K24P 0867
Reg. No. : $\qquad$
Name : $\qquad$
II Semester M.Sc. Degree (C.B.S.S. - Supple. (One Time Mercy Chance)/ Imp.) Examination, April 2024
(2014 to 2022 Admissions)
PHYSICS
PHY 2C07 : Mathematical Physics - II
Time : 3 Hours


Max. Marks : 60

SECTION - A

Answer both the questions (Either a or b).

1. a) i) Write down the Laplace equation in spherical polar coordinates and find its solution.
ii) Solve the following boundary value problem $\frac{\partial u}{\partial x}=4 \frac{\partial u}{\partial y}$, given $u(0, y)=8 e^{-3 y}$, by the method of separation of variables.

OR
b) i) What is Geometric series ? Under what condition a geometric series is convergent, divergent or oscillatory?
ii) State and explain any three methods for testing the convergence or divergence of a series.
2. a) i) State and prove Schur's lemmas for the irreducible representations of a group.
ii) Deduce the orthogonality theorem of the irreducible representation of a group.

b) Write a short note on:
i) Special Unitary Group $\operatorname{SU}(2)$
ii) Abelian Group
iii) Orthogonality theorem for characters
iv) Rearrangement theorem.

## SECTION - B

Answer any four questions ( $\mathbf{1}$ mark for Part a, $\mathbf{3}$ marks for Part b, $\mathbf{5}$ marks for Part $\mathbf{c}$ ).
3. a) Show that the series 5-4-1+5-4-1+5-4-1+ $\qquad$ $\infty$ is oscillatory.
b) "The nature of an infinite series remains unaltered by addition or removal of finite number of terms". Justify.
c) Apply D' Alembert's ratio test to show that the series
i) $\frac{n^{2}}{2^{n}}$ converges and
ii) $\frac{2^{n}}{n^{3}}$ diverges.
4. a) How many irreducible representations are possible for the $\mathrm{C}_{3 v}$ point group.
b) Illustrate the method of splitting partial differential equation into ordinary differential equations by taking Helmholtz equation as example.
c) Solve the following equation $\frac{\partial^{2} z}{\partial x^{2}}-2 \frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0 b y$ the method of separation of variables.
5. a) Write any one property of a Green's function.
b) What is Green's function? State and explain its symmetry property.
c) Explain the solution of Poisson's equation $\nabla^{2} \phi=-\frac{\rho(r)}{}$ using Green's function. CENTRAL LIBRA $\epsilon_{0} Y$
6. a) Define a Cyclic group.
b) Show that three cube root of unity $\left(1, \omega, \omega^{2}\right)$ where $\omega^{3}=1$, form an Abelian finite group under multiplication.
c) "The symmetry transformations of an equilateral triangle form a group". Comment.
7. a) What is meant by self-reciprocal with respect to a Fourier Transform?
b) Explain the convolution theorem on Fourier transform.
c) Find the Fourier transform of the function $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{ll}l+\frac{\mathrm{x}}{\mathrm{a}}, & -\mathrm{a}<\mathrm{x}<0 \\ l-\frac{\mathrm{x}}{\mathrm{a}}, & 0<\mathrm{x}<\mathrm{a} \\ 0, & \text { otherwise }\end{array}\right\}$.
8. a) Find the Laplace transform of $\mathrm{e}^{-\mathrm{at}}$.
b) Calculate the inverse Laplace transform of $F(s)=\frac{\log (s+a)}{\log (s+b)}$.
c) State and prove first and second shifting property of Laplace transform.


