K22P 1589

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
PHYSICS
PHY 1C02 : Classical Mechanics

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **both** questions (either **a** or **b**). **Each** question carries **12** marks. **(2×12=24)**

1. a) Write down the Lagrangian for a symmetric trilinear CO_2 and obtain the normal mode frequencies of oscillations. Explain the physical oscillations each of these frequencies represent. Choose mass of carbon atom to be M and that of oxygen atom to be m.

OR

b) Demonstrate that the Schrödinger equation for a quantum mechanical particle reduces in the classical limit to the corresponding Hamilton-Jacobi equation.

2. a) Explain the classical scattering in a central force potential V(r) and derive the Rutherford formula for scattering cross-section.

OR

b) State Hamilton's principle and derive Euler-Lagrange equations of motions using calculus of variations.

SECTION – B

Answer **any four** questions. (**1** mark for Part **a**, **3** marks for Part **b**, **5** marks for Part **c**) **(4×9=36)**

3. a) Define equilibrium points of a potential and explain how they are classified.

b) Explain normal modes of oscillations.

c) Can a particle of mass m experiencing a potential $V(r) = \frac{l^2}{2mr^2} - \frac{GMm}{r}$ have stable equilibrium points ? If yes, find the points and the frequency of small oscillations about the stable points. Here the constants l, G, M are positive numbers and the coordinate $r \geq 0$.

4. a) Explain what is a cyclic coordinate. Provide one example.

   b) Explain the type of constraints and the number of degrees of freedom for the following systems in three dimensional space.

      i) N particles moving on a cylinder whose radius R change in time.

      ii) A particle moving inside a cubical box with fixed edges.

   c) Write down the Lagrangian for a pendulum and obtain its Euler-Lagrange equations of motion. What form does the Lagrangian take when this is a simple pendulum ?

5. a) Explain any situation where a Hamiltonian method have an advantage over a Lagrangian method.

   b) Write down the Hamiltonian for a simple harmonic oscillator in one dimension and plot its phase space trajectory. What is the phase space trajectory if it is a damped oscillator ?

   c) i) Write down the Hamiltonian and obtain the Hamilton's equations of motion for a charged particle moving in an electromagnetic field.

      ii) Obtain the Lagrangian for the above from the Hamiltonian.

6. a) Write the generating function of an identity canonical transformation and demonstrate it.

   b) Hamiltonian for a particle is $H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{1}{2}mw^2(x^2 + 3y^2 + z^2)$. Find out which of the components of angular momentum vector L are conserved.

   c) For what value of the constant α does the transformation $Q = \frac{p}{2q}$ and $P = -\frac{\alpha q^2}{2}$ becomes a canonical transformation $(q, p) \rightarrow (Q, P)$? Apply this canonical transformation to a simple harmonic oscillator and find the Hamiltonian (Kamiltonian) in coordinates (Q, P).

7. a) Explain the relevance of Hamilton's characteristic function in Hamilton-Jacobi formalism.

   b) Explain how Hamilton-Jacobi method helps to solve a problem in mechanics.

   c) Solve simple harmonic oscillator using the method of action-angle variables.

8. a) A particle is moving on the surface of rigid body that is rotating with constant angular velocity ω. If the force acting on the particle measured from a space coordinate system $F_s = 0$. What is the acceleration of the particle at position r_b, as measured in the body system ?

b) Write down the Euler equations for an object that is symmetric about one axis and describe its motion qualitatively.

c) The moment of inertia tensor for a rigid body in a certain coordinate system is given by the matrix

$$\begin{pmatrix} 8 & -3 & -3 \\ -3 & 8 & -3 \\ -3 & -3 & 8 \end{pmatrix}$$

Find the moment of inertia tensor in the principal axes coordinate system.