K22P 1588

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS-Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
PHYSICS
PHY1C01 : Mathematical Physics – I

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **both** questions. (either **a** or **b**), **each** question carries **12** marks. **(2×12=24)**

1. a) Express the unit vectors in spherical polar coordinate system in terms of the unit vectors in Cartesian coordinates.

OR

b) Diagonalize the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

2. a) Discuss the Laurent series. Find the Laurent series of the function $f(z) = \frac{1}{1-z^2}$ with centre at z = 1.

OR

b) Deduce the relation $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and hence show that $\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin m\pi}$. Given that $\int_0^\infty \frac{y^{m-1}}{(1+y)} dy = \frac{\pi}{\sin m\pi}$.

SECTION – B

Answer **any four** questions, Part **a** carries **1** mark, Part **b** carries **3** marks and Part **c** carries **5** marks. **(4×9=36)**

3. a) If R is an orthogonal matrix, show that detR = ±1.

b) Show that the product of two orthogonal matrices is orthogonal.

c) Find the most general 2×2 orthogonal matrix.

4. a) Show that $\nabla \times \vec{r} = 0$.

   b) Resolve the cylindrical unit vectors into their Cartesian components.

   c) Obtain the Laplacian operator in cylindrical coordinates.

5. a) Comment on the eigenvalues of an anti Hermitian matrix.

   b) Show that the eigenvectors of a unitary matrix is unimodular.

   c) Consider the matrices $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. Verify whether they can be simultaneously diagonalized, and find the common eigenvectors of the two matrices.

6. a) Evaluate $\delta_{ik}\delta_{kl}$.

   b) Evaluate all the components of Levi civita tensor $\in_{ijk}$ in three dimensions, if $\in_{123} = 1$.

   c) Show that for Levi civita tensor, $\in_{ijk} \in_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$.

7. a) Write down the generating function for the Legendre polynomials.

   b) Obtain $P_1(x)$ and $P_2(x)$ from the generating function.

   c) Show that $P'_{n+1}(x) - P'_{n-1}(x) = (2n + 1)P_n(x)$.

8. a) Develop the Taylor expansion for $\ln(1 + z)$.

   b) Find the analytic function $w(z) = u(x, y) + iv(x, y)$ if $u(x, y) = x^3 - 3xy^2$.

   c) Find the residue of $f(z) = \dfrac{e^z}{z^2 + a^2}$ at its singularities.