



K23U 2369

Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/
Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B09MAT: Vector Calculus

Time : 3 Hours

Max. Marks : 48



PART – A

(Short Answer Questions)

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Find the parametric equation for the line through $(3, -4, -1)$ parallel to the vector $v = i + j + k$.
2. Find the distance from the point $(2, -3, 4)$ to the plane $x + 2y + 2z = 13$.
3. Find the gradient of the function $f(x, y) = xy^2$ at the point $(2, -1)$.
4. Evaluate $\int_C (x + y)ds$, where C is the straight line segment $x = t, y = 1 - t, z = 0$ from $(0, 1, 0)$ to $(1, 0, 0)$.
5. Define Divergence Theorem.

PART – B

(Short Essay Questions)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the length of the portion of the curve $r(t) = 4\cos t i + 4\sin t j + 3t k, 0 \leq t \leq \frac{\pi}{2}$.
7. Find the curvature of $r(t) = 3\sin t i + 3\cos t j + 4t k$.

P.T.O.



8. Find the directions in which $f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases more rapidly at $(1, 1)$.
9. Find the plane tangent to the surface $z = x \cos y - ye^x$ at $(0, 0, 0)$.
10. Find the work done by the force field $F = xi + yj + zk$ in moving an object along the curve C parametrized by $r(t) = \cos(\pi t)i + t^2j + \sin(\pi t)k$, $0 \leq t \leq 1$.
11. Find the scalar potential of the vector field $F = 2xi + 3yj + 4zk$.
12. Find the Curl of $F = (x^2 - z)i + xe^{zj} + xyk$.
13. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.
14. Find the Divergence of the vector field $F = (y^2 - x^2)i + (x^2 + y^2)j$.
15. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.
16. Evaluate $\int_C y^2 dx + x^2 dy$, $C : x^2 + y^2 = 4$.

PART - C
(Essay Questions)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find the angle between the planes $2x + 2y + 2z = 3$, $2x - 2y - z = 5$.
18. Find the unit tangent vector of the curve
 $r(t) = \sin t i + (3t^2 - \cos t)j + e^t k$, at $t_0 = 0$.
19. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $(1, 1, 0)$ in the direction of $v = 2i - 3j + 6k$.
20. Verify Green's theorem for $F = -yi + xj$ over the circle
 $C : a \cos t i + a \sin t j$, $0 \leq t \leq 2\pi$.



- 21. Verify Divergence theorem for $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.
- 22. Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point $(2, 1, 0)$.
- 23. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1, y = 1, z = 1$.

PART – D
(Long Essay Questions)

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

- 24. Find the curvature and torsion of the curve
 $r(t) = (\cos t + t \sin t)i + (\sin t - t \cos t)j, t > 0$.
- 25. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
- 26. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral
 $\int ydx + xdy + 4dz$ over any path from $(1, 1, 1)$ to $(2, 3, -1)$.
- 27. Find the center of mass of a thin hemispherical shell of radius a and constant density δ .