K24U 1637

Reg. No.: ....

Name : ....

Second Semester B.Sc. Degree (CBCSS – OBE-Regular/Supplementary/ Improvement) Examination, April 2024
(2019 Admission Onwards)
CORE COURSE IN STATISTICS
2B02STA : Probability Theory and Mathematical Expectation

Time : 3 Hours Max. Marks : 48

PART – A

Short answer. Answer **all** questions. **1** mark **each**.

1. Define Random experiment.
2. What do you mean by mutually exclusive events ?
3. Define probability mass function.
4. Define random variable.
5. Define mathematical expectation of a random variable.
6. Show that $V(aX+b) = a^2V(X)$. **(6×1=6)**

PART – B

Short essay. Answer **any 7** questions. **2** marks **each**.

7. Define the classical definition of probability.
8. Define field and sigma field.
9. The joint probability density function of two dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} 2, & 0 < x < 1, 0 < x < 1, 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$
   Find the marginal density function of X and Y.

P.T.O.

10. Distinguish between discrete and continuous random variable.

11. Define skewness and kurtosis of a continuous random variable.

12. Show that expectation of sum of random variable is equal to the sum of expectation of random variables.

13. Compute mean and variance of the random variable X having probability density function $f(x) = \begin{cases} \dfrac{1}{2\sqrt{x}}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

14. Define covariance between two random variables X and Y. If $\text{cov}(X, Y) = 2$, then find $\text{cov}(X + 8, Y = 6)$ and explain your answer.

15. Define characteristic function of a random variable. Find the value of $\phi_x(t)$ at $t = 0$. **(7×2=14)**

PART – C

Essay. Answer **any 4** questions. **4** marks **each**.

16. State and prove addition theorem of probability for two events.

17. Two balls are drawn without replacement from an urn containing 5 white and 3 black balls. Find the probability that

    i) both balls are of the same colour,

    ii) at least one ball is white.

18. The probability density function of a random variable X is given as follows.

X	0	1	2	3	4	5
P(X)	k^2	$\frac{k}{4}$	$\frac{5k}{2}$	$\frac{k}{4}$	$2k^2$	k^2

Find :

a) k

b) Write down the distribution of x.

19. Given, $f(x) = \begin{cases} k(x+2), & 1 < x < 5 \\ 0, & \text{elsewhere} \end{cases}$

Find the value of k and hence find the probability density function of $Y = X^2$.

20. Establish the relation between raw and central moments.

21. Demonstrate two properties of probability generating function of a random variable X. **(4×4=16)**

PART – D

Essay. Answer **any 2** questions. **6** marks **each.**

22. i) State Baye's theorem.

ii) The probabilities of A, B and C becoming managers of a company are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$ respectively. The probability that the bonus scheme will introduced if A, B and C become managers respectively are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively. If the bonus scheme has been introduced, what is the probability that managers appointed was A.

23. Two random variable X and Y have the following joint probability density function ;

$$f(x, y) = \begin{cases} 2-x-y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find :

i) Marginal probability density function of X and Y.

ii) Conditional density functions.

24. i) Define conditional expectation of X given Y and Y given X.

ii) Show that $E\left[E[X/Y]\right] = E[X]$.

25. If a random variable having the probability density function $f(x) = q^{x-1}p$, $x = 1, 2, 3, \ldots, p + q = 1$. Find the moment generating function and hence find its mean and variance. **(2×6=12)**