

Reg. No.:....

Name:

V Semester B.Sc. Degree (C.B.C.S.S.-O.B.E.-Regular/Supplementary/ Improvement) Examination, November 2023 (2019 – 2021 Admissions) CORE COURSE IN MATHEMATICS 5B06 MAT: Real Analysis – I

Time: 3 Hours Max. Marks: 48

PART - A

Answer any 4 questions. They carry 1 mark each.

 $(4 \times 1 = 4)$

- 1. State Triangle Inequality.
- 2. Find $\lim_{n \to \infty} \left(1 + \frac{1}{2n}\right)^n$
- 3. Define m-tail of a sequence.
- 4. Define continuity of a function at a point.
- 5. Define Rearrangement of the series.

PART - B

Answer **any 8** questions from among questions **6** to **16**. These questions carry **2** marks **each**. (8×2=16)

- 6. Determine the set A of $x \in R$ such that |2x + 3| < 8.
- 7. If $a \in R$ and $a \neq 0$ then show that $a^2 > 0$.
- 8. Discuss the convergence of $\lim \left(\frac{n}{2^n}\right)$
- 9. Find the limit of the sequence whose terms are given by $x_1 = 8$, $x_{n+1} = \frac{x_n}{2} + 2$ for $n \in \mathbb{N}$.
- 10. State Monotone Convergence Theorem.
- 11. Define subsequence of a sequence with an example.
- 12. State Alternating Series test.

K23U 2366



- 13. Define convergent Series.
- 14. If $\sum a_n$ with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n}$ always convergent. Justify.
- 15. Show that $f(x) = \frac{1}{x}$ defined on $A = (0, \infty)$ is unbounded on A.
- 16. State Boundedness Theorem.

Answer **any 4** questions from among questions **17** to **23**. These questions carry **4** marks **each**. (4×4=16)

- 17. Show that cosine function is continuous on R.
- 18. Discuss the convergence of $\sum_{n=0}^{\infty} r^n$, $r \in \mathbb{R}$, |r| < 1.
- 19. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$.
- 20. Discuss the convergence of the sequences
 - a) $((-1)^n)$ and
 - b) (n).
- 21. Show that Cauchy sequence of real numbers is bounded.
- 22. State and prove Archimedean property.
- 23. If a and b are positive real numbers, $a \neq b$ then show that $\sqrt{ab} \leq \frac{(a+b)}{2}$.

Answer **any 2** questions from among questions **24** to **27**. These questions carry **6** marks **each**. (2×6=12)

- 24. State and prove density theorem of rational numbers in R.
- 25. State and prove Squeeze theorem for sequences. Hence find $\lim \left(\frac{\sin n}{n}\right)$.
- 26. Discuss the convergence of

a)
$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} = NTRAL LIBRARY$$

b)
$$\sum_{n=1}^{\infty} \frac{\left(\cos n\right)}{n^2}.$$

- 27. Discuss the continuity of
 - a) Dirichlet's function
 - b) Thomae's function.