K21P 4196

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)
Examination, October 2021
(2018 Admission Onwards)
PHYSICS
PHY1C01 : Mathematical Physics – I

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **both** questions, either (**a**) or (**b**). **Each** question carries **12** marks.

1. a) Define eigen values and eigen vectors of a square matrix. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

OR

b) Obtain the series solution to the linear oscillator equation $y'' + \omega^2 y = 0$ using Frobenius' method.

2. a) Prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

OR

b) Write Laguerre ordinary differential equation and Laguerre polynomial $L_n(x)$. Obtain Rodrigues' formula for Laguerre polynomials. Deduce first three Laguerre polynomials. **(2×12=24)**

SECTION – B

Answer **any four** (**1** mark for Part '**a**', **3** marks for Part '**b**', **5** marks for Part '**c**').

3. a) Define divergence of a vector field.

b) Resolve the circular cylindrical unit vectors into their Cartesian components.

c) If $\vec{F} = \left(x^2 + y^2 + z^2\right)^n \left(\hat{e}_x x + \hat{e}_y y + \hat{e}_z z\right)$, find the scalar potential of $\vec{F}$.

P.T.O.

4. a) Define contravariant tensor.

   b) Explain three dimensional Levi-Civita symbol of tensors.

   c) Define the terms orthogonal matrix, hermitian matrix and unitary matrix. Give examples in each case.

5. a) What do you mean by singular point of an ordinary differential equation ?

   b) What do you mean by Wronskian of an ordinary differential equation ? Discuss the linear independence of solutions of an ordinary differential equation in terms of Wronskian.

   c) Solve the inhomogeneous ordinary differential equation $(1-x)y'' + xy' - y = (1-x)^2$.

6. a) Give an example for an analytic function.

   b) Define poles and residues of a complex function. Find the residue of $f(z) = \frac{1}{\sin z}$ at $z = 0$.

   c) State and prove Morera's theorem for a complex function.

7. a) Define beta function.

   b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

   c) What is the relation between beta and gamma functions ? Prove that

$$\int_0^{\frac{\pi}{2}} \cos^{\frac{1}{2}}\theta \, d\theta = \frac{(2\pi)^{\frac{3}{2}}}{16\left(\Gamma\left(\frac{5}{4}\right)\right)^2}.$$

8. a) Write the first three Legendre polynomials.

   b) Define spherical Bessel function. Write the expression for $j_2(x)$.

   c) For Leguerre polynomials $L_n(x)$, prove that $xL_n'(x) = nL_n(x) - nL_{n-1}(x)$. **(4×9=36)**
