Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B08 MAT : Differential Equations and Laplace Transforms

Time : 3 Hours

Max. Marks : 48

PART – A
(Short Answer)

Answer **any four** questions from this Part. **Each** question carries **1** mark.　　**(4×1=4)**

1. Solve the differential equation $y' = 1 + y^2$.

2. Check whether the equation $-y\,dx + x\,dy = 0$ is exact.

3. Give an example of a non-homogeneous differential equation.

4. Solve $y'' - y = 0$.

5. State the linearity property of the Laplace transform.

PART – B
(Short Essay)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the order and degree of the differential equation $\dfrac{d^2y}{dx^2} + \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 0$.

7. Prove that e^x is an integrating factor of $\sin y\,dx + \cos y\,dy = 0$ and solve it.

P.T.O.

8. Find the orthogonal trajectories of the curve $y = ce^{-x}$.

9. State the existence theorem of first order differential equations.

10. Solve the initial value problem $y'' - y' - 2y = 0$, $y(0) = -4$, $y'(0) = -17$.

11. Check whether the solutions x^2 and $x^2 \ln x$ are linearly independent.

12. Find the Laplace transform of $a + bt + ct^2$.

13. Solve $y'' + 25\ y = 0$.

14. Find the inverse Laplace transform of $\dfrac{12}{(s-3)^4}$.

15. Write the standard form of Euler Cauchy equation. Give an example.

16. Solve $2x \tan y\ dx + \sec^2 y\ dy = 0$.

PART – C

(Essay)

Answer **any four** questions from this Part. **Each** question carries **4** marks.　　**(4×4=16)**

17. Find the general solution of $y' - y = e^{2x}$.

18. Solve $y'' + 2y' + y = x^2$.

19. Let $f(t) = t\ \sin wt$, find the Laplace transform of $f(t)$.

20. Check for exactness and solve the initial value problem, $ye^x dx + (2y + e^x)dy = 0$, $y(0) = -1$.

21. Solve $y' = (y + 4x)^2$.

22. Solve $(\cot y + x^2)dx = x\csc^2 y\ dy$.

23. Solve $y'' + y = \sec x$.

PART – D
(Long Essay)

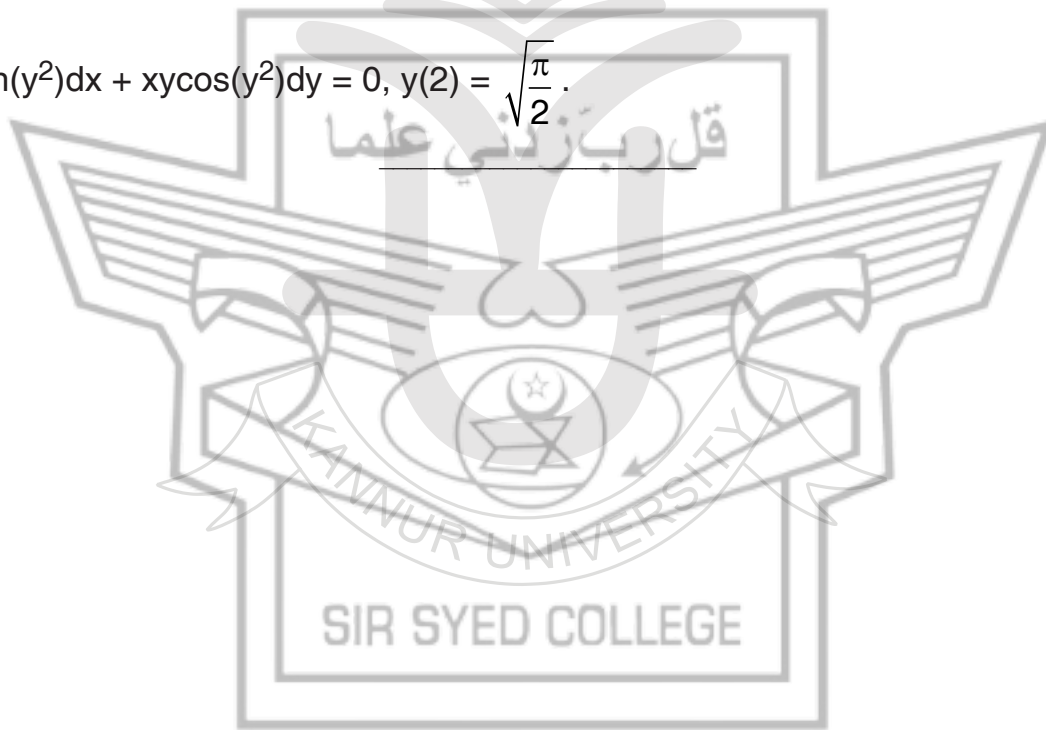Answer **any two** questions from this Part. **Each** question carries **6** marks.   (2×6=12)

24. Using Laplace transforms, solve the integral equation, $y(t) = 1 - \int_0^t (t - \tau) \, y(\tau) d\tau$.

25. Solve $4x^2D^2 + 24xD + 25 \, y = 0$, $y(1) = 2$, $y'(1) = -6$.

26. Solve $y'' + 2y' + 5y = 1.25e^{0.5x} + 40 \cos 4x - 55 \sin 4x$, $y(0) = 0.2$, $y'(0) = 60.1$.

27. Find an integrating factor and solve the initial value problem

$2\sin(y^2)dx + xy\cos(y^2)dy = 0$, $y(2) = \sqrt{\dfrac{\pi}{2}}$.